# Written Exam for the B.Sc. or M.Sc. in Economics autumn 2011-2012 

## Corporate Finance and Incentives

Final Exam/ Elective Course/ Master's Course
$24^{\text {th }}$ February 2012
(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. Each problem approximately weights the same in the final grade. A problem can consist of different sub questions that do not necessarily have equal weight. Please provide intermediate calculations.

## Problem 1 (Various themes)

1. What are the assumptions underlying the CAPM and what is the main difference between the assumptions of CAPM and factor models?
2. Explain how the minimum-variance portfolio and the tangency portfolio are found in CAPM.
3. What are the Capital Market Line (CML) and the Securities Market Line (SML) and how are they derived?
4. In a binomial model, what is the probability of the good state (up) in below diagram, if the risk free rate is $10 \%$ ?

5. What is the most common input for the risk-free return when practitioners (Wall-Street) use valuation models, and why does this make good sense?
6. Explain the put-call parity and show how to derive it.
7. Explain three shortcomings of the Internal Rate of Return (IRR) method.
8. Explain the three sources of value creation for shareholders (PV of unlevered cash flow, PV of financing subsidies and transfers from existing debt holders).

## Problem 2 (No comparable firms)

Your company considers investing $\$ 1$ million in a project, of which there are no comparable companies or projects. Fortunately you prepared well and know the following expected cash flows contingent on states of the world. The risk free rate is $5 \%$.

| State of the <br> world | Probability | Market return <br> $\mathbf{( \% )}$ | Incremental <br> cash flow in one <br> year | Net return on <br> invested capital |
| :--- | :---: | :---: | :---: | :---: |
| Boom | $50 \%$ | $35 \%$ | $\$ 1,500,000$ | $50 \%$ |
| Recession | $40 \%$ | $5 \%$ | $\$ 1,200,000$ | $20 \%$ |
| Depression | $10 \%$ | $-40 \%$ | $\$ 800,000$ | $-20 \%$ |
| Expected return |  | $15.5 \%$ | $\$ 1,310,000$ | $31 \%$ |

1. Find the NPV of this project under the assumption that CAPM holds.
2. What is the correct beta of this project?
3. What is the relation between the return beta and the correct beta?

The risk-adjusted discount rate method: $P V=\frac{E(\tilde{C})}{1+r_{f}+\beta\left(\bar{R}_{T}-r_{f}\right)}$
The certainty equivalent method: $P V=\frac{E(\tilde{C})-b\left(\bar{R}_{T}-r_{f}\right)}{1+r_{f}}$, where $b=\frac{\operatorname{cov}\left(\tilde{C}, \tilde{R}_{T}\right)}{\sigma_{T}^{2}}$

## Problem 3 (Fixed Income)

Consider a market with four bonds:

| Type of bond | Coupon | Price |
| :--- | :--- | :--- |
| 1-year bullet bond | $3 \%$ | 99.04 |
| 2-year serial bond | $5 \%$ | 100.51 |
| 3-year annuity bond | $6 \%$ | 101.77 |
| 4-year annuity bond | $6 \%$ | 101.52 |

The bonds pay exactly one, two, three and four years from today.

1. Find the four discount factors, the four spot rates (yield to maturity (YTM) for the zero coupon bonds) and the four forward rates.
2. Find the YTM for the four bonds.
3. Find the price and YTM for a newly introduced 4 -year bullet bond with a coupon of $8 \%$.
4. Explain (no calculations needed) if you expect the first derivative of YTM with respect to the coupon rate $(\partial Y T M / \partial i)$ to be close to zero or close to one. Why?
5. Find the Macaulay duration for the new 4 -year bullet bond. Explain what this duration tells us and why the duration increases when the coupon rate decreases.
6. Rank an annuity, bullet and serial loan in regards to duration, all other things being equal, and explain how you came to this conclusion.

Macaulay duration: $D(c, r)=\frac{1}{P V(c, r)} \sum_{t=1}^{T} t \frac{c_{t}}{(1+r)^{t}}$

## Problem 4 (Real investments and real options)

Assume that a copper mining company currently is bound to a contract with the union, stating that laying off employees requires a two-year notice period. The company borrows at $5 \%$, which is also assumed to be the risk free rate, and the required return on equity is $15 \%$. The company has a constant debt to equity ratio of $50 / 50$.

The company extracts $1,000,000$ kilos ( 1,000 ton) of copper per year at a cost of $\$ 6$ per kilo, and is expected to do this perpetually (for simplicity sake). The price of copper fluctuates and can each future year take one of two values; with $50 \%$ probability the price is $\$ 5$ per kilo and with $50 \%$ probability it is $\$ 9$ per kilo (just like flipping a coin each and every year).

1. What is the PV of the mining company?
2. If the company could lay off people without notice period (instantly), what would then be the value of the mining company? Explain how this adds value to the mining company.
3. Assume now that the government and the company writes a contract, which legally obligates the mining company to sell its copper at $\$ 7$ per kilo and the government to buy the copper at $\$ 7$ per kilo (no options) perpetually. What is the value - if any - of such a contract to the mining company, compared to the situation in sub question 1 (2 years laying off notice)? Explain.
